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AM-GM Any Baby's BMs

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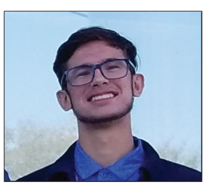

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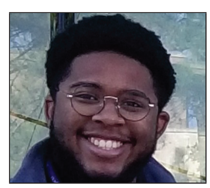

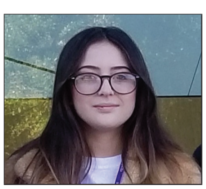

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1 Introduction

Reusable diapers are growing in popularity due to new and improved designs. The total cost associated with reusable diapers is the sum of the upfront cost of purchasing the diapers and the reoccurring cost of cleaning the diapers. With a nice application of the AM-GM inequality we show that the minimum cost to the consumer is twice the geometric mean of the upfront and cleaning costs plus the cost of a single reusable diaper. Moreover, the minimum is obtained when the upfront cost is equal to the total cleaning costs plus the cost of a single reusable diaper. We conclude with an analysis of our reusable diaper model and a cost comparison with disposable diapers.

The arithmetic mean (AM) and geometric mean (GM) of $a_1, a_2, \ldots, a_n \geq 0$, are defined as follows:

$$\frac{a_1 + a_2 + \cdots + a_n}{n} \tag{AM}$$

$$\sqrt[n]{a_1 a_2 \cdots a_n} \tag{GM}$$

Theorem 1 (The AM-GM inequality)**.** *If $a_1,\ a_2,\ \ldots,\ a_n \geq 0$ then*

$$a_1 + a_2 + \cdots + a_n \geq n \sqrt[n]{a_1 a_2 \cdots a_n}.$$

Moreover, equality occurs if and only if $a_1 = a_2 = \cdots = a_n$.

We use the term BM to refer to either a bowel movement or bladder movement. In our model we make the assumption that the baby's diaper is changed after each and every BM.

2 The model

The independent variable for our model is the number of diapers purchased by the consumer which we denote by x. Our model makes the following two dubious assumptions: (†) $x - 1$ diapers must be dirty when we launder them and (‡) all $x - 1$ diapers can be washed in one load of laundry. The total cost can be computed in terms of three parameters:

p = price of a reusable diaper.

d = number of BMs until the child is potty trained.

w = cost to wash one full load of diapers.

The number of total washes needed is $\frac{d}{x-1}$ washes. The total cleaning costs are then $\frac{d}{x-1} \cdot w$ dollars. The upfront cost of purchasing the diapers is $p \cdot x$ dollars. The sum of the upfront cost and cleaning costs gives the total cost function for our model

$$C(x) = px + \frac{dw}{x-1}.$$

If the consumer only purchases two reusable diapers, then each time a diaper is soiled it must be washed - this gives a low upfront cost but large cleaning costs. Alternatively, the consumer could buy a new reusable diaper every time that a previous diaper is soiled. This requires little cleaning costs (one wash at the end, eww) but a larger upfront cost. In particular, minimizing the sum of the upfront costs and the cleaning costs does not occur in either situation. However, this suggests a closed domain for optimizing our cost function $C(x)$, namely $[2, d]$.

Exercise 1. *Find the absolute minimum value of $C(x)$ on the closed interval $[2, d]$. Find a formula for the minimum cost in terms of the parameters p, d and w.*

We provide a solution to the exercise using the AM-GM inequality. For all $x > 0$,

$$\begin{aligned} C(x) = px + \frac{dw}{x-1} &= p + p(x-1) + \frac{dw}{x-1} \\ &\geq p + 2\sqrt{p(x-1)\frac{dw}{x-1}} \\ &\geq p + 2\sqrt{pdw}. \end{aligned}$$

Equality occurs only when $p(x-1) = \frac{dw}{x-1}$. Solving the previous equation for x and rationalizing the denominator we find that the minimum cost occurs at $x = 1 + (1/p)\sqrt{pdw}$ diapers. In particular, the following two formulas can be used by a consumer that has estimated the values of the parameters p, d and w to minimize their costs:

$$\left\{ \begin{array}{c} \text{Recommended number of reusable diapers} = 1 + (1/p)\sqrt{pdw}, \\ \\ \text{Associated minimum cost} = p + 2\sqrt{pdw}. \end{array} \right\} \quad (1)$$

3 The true minimum cost

Our model assumed that x was a continuous variable. However, x can only take on integer values. Since the cost function is concave up on $[2, d]$, the true minimum cost occurs at either the smallest integer greater than $1 + (1/p)\sqrt{pdw}$ or the greatest integer less than $1 + (1/p)\sqrt{pdw}$. To find the true minimum cost we evaluate the cost function $C(x) = px + \frac{dw}{x-1}$ at these two integers and keep the smaller of the two values. That is,

$$p + \min\left(p\left\lceil 1/p\sqrt{pdw}\right\rceil + dw/\left\lceil 1/p\sqrt{pdw}\right\rceil, p\left\lfloor 1/p\sqrt{pdw}\right\rfloor + dw/\left\lfloor 1/p\sqrt{pdw}\right\rfloor\right).$$

The next table collects together our recommendations for consumers based on the previous formula for a wide range of values of p and w.

$d = 8480$	$w = 0.50$	$w = 0.70$	$w = 0.90$	$w = 1.10$
$p = 5.00$	\$296.21 30 diapers	\$349.59 35 diapers	\$395.69 40 diapers	\$436.93 44 diapers
$p = 10.00$	\$421.90 22 diapers	\$497.33 25 diapers	\$562.57 29 diapers	\$620.90 32 diapers
$p = 15.00$	\$519.41 18 diapers	\$611.80 21 diapers	\$691.83 24 diapers	\$763.12 26 diapers
$p = 20.00$	\$602.67 16 diapers	\$709.18 18 diapers	\$801.60 21 diapers	\$884.00 23 diapers

Figure 1: Number of recommended diapers and associated true minimum costs with $d = 8480$.

Research suggests that females potty train more quickly than males [1]. Moreover, potty training rates vary widely geographically and culturally [1]. We roughly estimate d, using numbers from [1], to be 8480 BMs for an average child in the United States of America.

4 Reusable or disposable?

In this section we derive an equation that can be used to determine how many soiled diapers are required to make reusable diapers a more cost effective choice than disposable diapers.

Let p_d denote the cost of a disposable diaper and d_e denote the number of soiled diapers needed to make reusable diapers the more cost effective option. The total cost to the consumer for using only disposable diapers is $p_d{\cdot}d_e$ dollars. Reusable diapers become more cost effective precisely when this cost is equal to the total minimum cost involved in adopting reusable diapers. That is, when $p_d d_e = p + 2\sqrt{p d_e w}$. Solving for d_e we find that

$$d_e = \frac{p\left(2w + p_d + 2\sqrt{w(w + p_d)}\right)}{p_d^2}. \tag{2}$$

A consumer that has estimated p_d, p and w can use equation (2) to determine how many diapers a child must soil for reusable diapers to be the better choice from a cost perspective.

With an application of the AM-GM inequality we minimized the cost associated to adopting reusable diapers. Our main goal is to make this information accessible and easy to interpret by anyone. Our recommendations allow consumers the ability to approximate costs based on their personal inputs and come to their own conclusions on whether or not it would benefit them to invest in reusable diapers.

Bibliography

[1] Thaman, L.A. and Eichenfield, L.F., *Diapering Habits: A Global Perspective.* Pediatric Dermatology Vol. 31 Suppl. 1 15–18, 2014. doi:10.1111/pde.12468