An Introduction to Exotic Options

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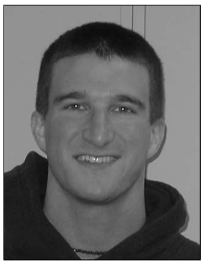

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Options or "privileges" as they were known in early 19th Century America actually appeared on the financial scene around the same times as stocks. Initially, there were numerous problems with the trading of options. The terms of the contract were different from contract to contract, contracts had to be exercised in person, and there really was no secondary market to trade. Options were eventually standardized in regards to strike price, expiration, size, and other relevant contract terms. The problem of pricing options would also be solved in time. Fisher Black, Myron Scholes, and Robert Merton developed a model for pricing options that is known today as the Black-Scholes model. Finally, all problems seemingly fixed, the Chicago Board of Trade established the Chicago Board Options Exchange on April 26th, 1973. Today, the average daily options volume is around 4.6 million contracts compared to 200,000 contracts at the end of 1974.

An option gives the owner the right to buy or sell an asset at a specified price on or before a specified expiration date. The owner is not obligated to exercise the option. In fact, sometimes it is to the owner's benefit to simply do nothing and let the option expire. There are two basic types of options. A call option gives the investor the right to buy the underlying asset at a specified price on or before a specified expiration date. A put option gives the investor the right to sell the underlying asset a specified price on or before a specified expiration date. The strike price, set at the purchase of the option, is the price at which the owner can buy or sell the underlying asset.

Options are used mainly for hedging and speculating purposes. Those who use options for hedging are protecting themselves from adverse future price movements by paying an up front fee. For example, an investor that owns 1,000 shares of company X, that is currently at \$75, might want to hedge against a large price drop. The investor could purchase 10 put option contracts (100 shares each) with a strike price of \$70. If the price per option was \$250, then the total price of hedging would be \$2,500. If the stock price were to rise to

$85, then the investor would not exercise the option and would be out $2,500. However, if the price were to drop to $50, the investor would execute the option. The investor would sell at $70 and would end up losing $7,500 ($5,000 on the stock, $2,500 on the option). If the option was not purchased, the investor would have lost $25,000. One could argue that if the price goes up in this example that an investor with a put option will not make as much money as another individual who does not have the put option. This is true; however, most people who use options to hedge feel the price of the option is well worth the opportunity to minimize the risk in an investment. It is also important to note that, theoretically, a portfolio that contains a combination of assets and put options can minimize the risk of the portfolio to zero.

Many traders also see options as a tremendous speculating opportunity. Speculators are betting either the price will go up or the price will go down and will purchase a call or put option based on their prediction. Speculating with options can magnify gains greatly, but can magnify losses tremendously as well. For example, a speculator might have $8,000 available to invest. The stock the speculator wants to invest in is currently trading at $40 and it is the speculator's belief that the price will be going up. If the price of a call option is currently $1 per share and has a strike price of $45, then the speculator could purchase 80 option contracts (100 shares each) or the speculator could buy 200 shares of the stock. Suppose the stock goes up to $55. Then $3,000 (200 × $15) would have been made if the speculator had purchased the 200 shares of stock. However, the speculator would make $72,000 (8,000 × $10 − $8,000) if the options were bought instead of the stock. What happens if the stock drops to $35 instead? If the 200 shares of stock were bought, then the speculator would have lost $1,000 (200 × −$5). Unfortunately, if the options were purchased instead, the speculator would have lost $8,000 (the price of purchasing the options).

The two most widely used types of options are European and American options. European options have a set strike price that gives you the right to buy or sell at the expiration of the option. American options are similar to European options. They have a set strike price and a given expiration date, but they may be exercised at any time before the expiration date. The flexibility that comes with an American option also makes them more expensive than the European option. These options are the simplest types of options and are considered standard options. Exotic options are nonstandard options. These options are simply standard options that have been modified to be beneficial in different situations. A few of the most common exotic options are mentioned below.

Chooser Options

Also known as "as you like it" options, chooser options allow you to choose whether the option is a put or a call at a specific date. The underlying options in chooser options are usually both European and have the same strike price. More complex chooser options do not have the same strike price or time to maturity. These options will, consequently, be more expensive than the individual underlying put or call options, because the owner has the right to choose. Chooser options are most useful in hedging against a future event that might not occur. They are used often around major events such as elections and when large market movements are expected. The date at which the option holder must choose between the put and the call is often a few days after the major event takes place. For example, the passage of a bill in Congress is expected to positively affect a particular company. However, if the bill is not passed, the company might be affected negatively. Buying a chooser option allows a stockholder in the company to hedge against a possible loss.

Lookback Options

The payoff from a lookback call is the difference between the final price of the stock minus the lowest observed price of the stock during a specific time period. The payoff from the lookback put is the difference between the highest observed price of the stock minus the final price of the stock during a specific time period. These options have an obvious advantage over regular American options. The owner of the lookback option does not have to worry about when to exercise the option. The owner can simply look back over the life of the option and find when it was most advantageous to exercise the option. Consequently, lookback options are more expensive than other vanilla and exotic options. Investors are more likely to use a lookback option in a more volatile marketplace where it is more likely the payout will offset the high premium on the option.

Shout Options

A shout option is a European option where the holder of the option is allowed to "shout" once at any time before the expiration of the option. The owner receives whichever is largest, the difference between the stock price and exercise price at expiry or the difference between the stock price and exercise price at the time of the "shout." The ability to go back at some point and value the option makes a shout option similar to a lookback option except a shout option is considerably less expensive and rightfully so. The owner of a shout option can only lookback to one specific time in the life of the option. Consequently, investors are more likely to use shout options in situations where European options are needed than in situations where lookback options are used.

Asian Options

The payoff of an Asian option depends on the average price of the underlying asset during a specific time period. The payoff for an Asian call is the maximum of zero and the stock price at expiry minus the average stock price. The payoff for an Asian put is the maximum of zero and the average stock price minus the stock price at expiry. The expected payoff from Asian options is less than the expected payoff from a European option and therefore, is less expensive than European options. Asian options are most helpful when hedging against events that take place over a period of time. For example, using an Asian option to hedge against the exchange rate would be appropriate if a country knew they were to receive a steady cash flow of another currency over a longer period of time.

Barrier Options

The payoff of a barrier option depends on whether the underlying asset's price reaches a certain level during a certain period of time. There are many types of barrier options. Barrier options are first classified into knock-in and knock-out options. Knock-out options exist until they reach a certain level and cease to exist once the level is reached. Knock-in options fail to exist until they reach a certain level at which point they come into existence. The barrier level can be reached from above or from below and so these options can be classified further as either down or up. If the initial price is above the barrier level, then it is labeled down. Similarly, if the initial price is below the barrier level is labeled up. This gives us four types of barrier options: up-and-in, up-and-out, down-and-in, down-and-out. Furthermore, each of these options can be either a call or a put.

Barrier options can be viewed as European options that are not in existence either before a barrier is hit or after a barrier is hit. For this reason, barrier options are cheaper than European options. Barrier options are usually used by investors who are looking to hedge, but at a cheaper price. For example, a portfolio manager might want to hedge against a large drop in the price of the portfolio. The manager could protect against a large drop with a European put, but could do it for less money by obtaining a down-and-in put. The manager could simply set a price at which they were not willing to let the price of the portfolio fall below. This set price would be the barrier. The price would be approaching from above the barrier and the manager would not want the option to come alive until the barrier was reached, so the manager would purchase a down-and in put. In this case, the manager feels the cost difference between a down-and-in put and a European put option is worth the risk of a price drop somewhere between the starting portfolio price and the barrier.

Binary Options

Cash-or-Nothing Options

A cash-or-nothing call pays a fixed amount at expiration if the stock price at expiration exceeds the exercise price. Similarly, a cash-or-nothing put pays a fixed amount at expiration if the stock price is less than the exercise price at expiration. The exercise price for this particular option plays no role in determining the amount of the payoff, but simply determines whether the owner of the option receives the fixed payoff.

Asset-or-Nothing Options

An asset-or-nothing option is similar to a cash-or-nothing option with the difference being the payoff. In an asset-or-nothing option, the option owner receives the asset, instead of a fixed amount as in a cash-or-nothing option. The stock price must be greater than the exercise price at expiry for the owner of the option to receive the asset in a call otherwise the owner receives nothing. Similarly, the stock price must be less than the exercise price at expiry for the owner to receive the asset in a put option otherwise the owner receives nothing.

Note: A European call option is equivalent to a long position in an asset-or-nothing call and a short position in a cash-or-nothing call where the cash payoff is equal to the strike price. In addition, a European put option is equivalent to a short position in an asset-or-nothing put and a long position in a cash-or-nothing put where the cash payoff is equal to the strike price. Because a European option can be formed from short and long positions in binary options, binary options tend to be cheaper than European options.

A Mathematical Model for Stock Price

Although first described and named after English botanist Robert Brown, it was the French mathematician Bachelier who first used Brownian motion to model price movements of stocks and commodities. The main idea behind the Brownian motion model is to assume that stock price movement can be modeled using a normal random variable. However, there are two major flaws when using Brownian motion to model stock and commodity prices. First, if the price of a stock is a normal random variable, then it could become negative. Second, it is not reasonable to assume that the price difference over a fixed interval has the same normal distribution no matter what the beginning price might be. To eliminate these problems, the geometric Brownian motion model was introduced. The geometric Brownian motion model uses a lognormal random variable to model stock price movement. This effectively eliminates the possibility of negative stock prices and assumes the distribution depends on the percentage change in price whose probabilities do not depend on the present price.

An intuitive way of showing that a stock's price is a lognormal random variable follows:

$$S_n = \text{price at time } n$$

$$y_n = \frac{S_n}{S_{n-1}} \text{ (the percentage change)}$$

Then, the stock price at time n can be written as a product of every percentage change over the given time period:

$$S_n = y_n \cdot y_{n-1} \cdot \ldots \cdot y_1 \cdot S_0$$

Then, by taking the natural log of each side we get

$$\ln(S_n) = \ln(S_0) + \sum \ln y_i$$

and using the Central Limit Theorem the sum $\sum \ln y_i$ is found to be approximately normal. Therefore, $\ln(\frac{S_n}{S_0})$ is normal.

The exact model is:

$$\ln(\frac{S_T}{S_0}) \sim N\Big((\mu - \frac{\sigma^2}{2})T, \sigma^2 T\Big)$$

or

$$\ln(S_T) \sim N\Big(\ln(S_0) + (\mu - \frac{\sigma^2}{2})T, \sigma^2 T\Big)$$

Note: Deriving the parameters $(\mu - \frac{\sigma^2}{2})T$ and $\sigma^2 T$ is a more difficult process [1].

Risk Neutral Valuation

The average rate of return, μ, is not easily calculated, but is a vital part in the pricing of an option. Risk neutral valuation takes care of this problem by assuming that individuals are indifferent to risk, so the expected return on all securities is the risk free interest rate, denoted by r. The discounted payoff of the option will also be discounted at the risk free rate when using this valuation technique. Therefore, when using risk neutral valuation in the pricing of various options, μ can be replaced with r.

Pricing an Asset-or-Nothing Option Using Risk Neutral Valuation

Consider an asset-or-nothing call option which pays S_T if $S_T > K$ at the expiry of the option, and 0 otherwise. (Here K is the exercise price.) So the payoff function is

$$h(S_T) = \begin{cases} S_T \text{ if } S_T > K \\ 0 \text{ otherwise.} \end{cases}$$

Using the risk-neutral valuation, we write the discounted expected value of the payoff as $e^{-rT}E[h(S_T)]$, where r is the risk-free interest rate. The problem reduces to finding $E[h(S_T)]$. From our asset price model, we know that S_T has lognormal distribution, in other words, $S_T = e^X$ where X has normal distribution $N(\ln S_0 + (r - \sigma^2/2)T, \sigma^2 T)$ where S_0 is the initial asset price and σ is the volatility. Then the expected value is computed as

$$E[h(S_T)] = E[h(e^X)] = \int_{\ln K}^{\infty} e^x f(x) dx$$

where $f(x)$ is the density function for $N(\ln S_0 + (r - \sigma^2/2)T, \sigma^2 T)$. The above integral simplifies to $S_0 e^{rT} N(d_1)$ (see [3]), where

$$d_1 = \frac{\ln(S_0 e^{rT}/K) + \sigma^2 T/2}{\sigma\sqrt{T}}.$$

Therefore, the price of the asset-or-nothing call option is

$$e^{-rT}E[h(S_T)] = e^{-rT}\left(S_0 e^{rT} N(d_1)\right) = S_0 N(d_1).$$

References

[1] S. Ross, An Introduction to Mathematical Finance, Cambridge University Press (1999).

[2] V. Bhansali, Pricing and Managing Exotic and Hybrid Options, McGraw-Hill (1998).

[3] J. Hull, Options, Futures, and Other Derivatives, Prentice Hall (2003).

[4] R. Kolb, Futures, Options, and Swaps, Blackwell Publishers (2000).

[5] S. Shamah, A Currency Options Primer, Wiley (2004).