Student Projects

Demand with Celebrity Effects

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Suppose you have never tried Indian food, so you and some friends go to an Indian restaurant. One of your friends happens to be an avid gastronome, so, understandably, you decide to follow her lead and order the lamb curry, which she says is one of her favorites.

Now suppose that you are about to buy a sweater at a department store. You are in line to check out when you notice that a person much older than you is wearing the exact same sweater. You are unwilling to accept that you have such dated tastes and put the sweater back.

Each of the above situations demonstrates what I will refer to as a "celebrity effect." In the first situation, your friend had a positive celebrity effect on your preferences. In the second example, the older person had a negative celebrity effect. In this article, I introduce such effects into demand theory through what I call "conditional" demand curves.

Background

In economics, individuals (agents) are faced with a set of goods at certain prices. Each agent has an endowment that allows them to purchase these goods. The problem for the agent is then to maximize some objective function by purchasing and consuming the appropriate amount of these goods. That this maximization process is undertaken we will take for granted in this article, although an overview of traditional demand theory is recommended to the curious reader [1]. With this in mind, we will begin with some notation.

Definition. The set of goods an agent purchases is a vector in $\mathbb{R}^n$ denoted by

$$\mathbf{x} = (x_1, x_2, \ldots, x_n)$$

where each x_i is the amount of good i purchased.

Definition. The set of prices that an agent must pay for goods purchased is a vector in $\mathbb{R}^n$ denoted by

$$\mathbf{p} = (p_1, p_2, \ldots, p_n)$$

where each p_i represents the market price of good i.

Definition. Each agent has an endowment that allows her to purchase goods. This is denoted by w (for wealth). Note that w can change from agent to agent, but in the present context we simply refer to w for the "representative" agent.

Definition. Agents choose the number of each different good such that some objective function is maximized. The amount of each good varies according to the price of that good, the prices of the other goods, and the endowment. We call the resulting function a demand function and denote it by

$$x_i = x_i(\mathbf{p}, w).$$

We now place a restriction on consumer behavior. This restriction is called *Walras's Law*. In short, it says that an agent purchases enough goods in order to use all of their endowment:

$$\mathbf{x}(\mathbf{p}, w) \cdot \mathbf{p} = \sum_{j=1}^{n} x_j(\mathbf{p}, w)\, p_j = w$$

This may seem strange and overly strong of an assumption. However, notice that our definition of goods allows for enough generality to allow bonds, stocks, even charitable giving. We now make another assumption on behavior, namely that

$$\frac{\partial x_i}{\partial p_i}(\mathbf{p}, w) < 0$$

for each $i = 1, 2, \ldots, n$. This is the economic idea that people buy less of something at a higher price, all else equal.

Celebrity effects in positive and negative directions

To discuss the examples above, we must expand our discussion to multiple agents making choices concurrently rather than looking only at a "representative" agent. In order to include multiple agents, we consider a set of agents

$$\mathcal{A} = \{A_1, A_2, \ldots, A_k\}$$

where each A_i represents a separate agent. We now explicitly allow the endowment of each agent to differ, so that the endowment of agent A is w_A. We also alter the demand function.

Definition. Consider two agents $A, B \in \mathcal{A}$. The conditional demand for good i by agent A, given the purchase of an amount of good i by agent B, is denoted by

$$x_{A,i}(\mathbf{p}, w_A \,|\, x_{B,i}).$$

With this in mind, we can make the following definition.

Definition. Given two agents $A, B \in \mathcal{A}$, who choose to purchase bundles of goods from the same selections at the same available prices, we say that:

i. Agent B exhibits a *positive* celebrity effect in good i for agent A if

$$x_{A,i}(\mathbf{p}, w_A \,|\, x^*_{B,i}) > x_{A,i}(\mathbf{p}, w_A \,|\, x_{B,i}) \text{ whenever } x^*_{B,i} > x_{B,i}\,.$$

ii. Agent B exhibits a *negative* celebrity effect in good i for agent A if

$$x_{A,i}(\mathbf{p}, w_A \,|\, x^*_{B,i}) < x_{A,i}(\mathbf{p}, w_A \,|\, x_{B,i}) \text{ whenever } x^*_{B,i} > x_{B,i}\,.$$

iii. Agent B does not exhibit celebrity effects in good i for agent A if

$$x_{A,i}(\mathbf{p}, w_A \,|\, x^*_{B,i}) = x_{A,i}(\mathbf{p}, w_A \,|\, x_{B,i}) \text{ for all } x_{B,i}\,.$$

We can now derive a useful proposition concerning these sorts of demand functions.

Proposition. If an agent A has conditional demand for some good i for which celebrity effects are present with respect to the amount of i purchased by some other agent B, then there must be at least one other good j for which agent B's purchases affect those of agent A in the opposite direction as that of good i.

PROOF. We will look at two goods, but this could be expanded. We will also show only the case with an initial positive celebrity effect and show that there must be an offsetting negative effect, but the other direction is similar.

To begin, let there be two goods, good 1 and good 2, and two agents, A and B. Let agent A's demand for good 1 be given by

$$x_{A,1}(\mathbf{p}, w_A \,|\, x_{B,1}).$$

Now assume that agent B exhibits a positive celebrity effect in good 1 for agent A. Suppose that agent B increases consumption of good 1, but not of good 2 (suppose B's wealth goes up and B spends the entire increase on good 1). Then $x_{B,1}$ increases to $x^*_{B,1}$. Thus the quantity of good 1 purchased by agent A increases as well:

$$x_{A,1}(\mathbf{p}, w_A \,|\, x^*_{B,1}) > x_{A,1}(\mathbf{p}, w_A \,|\, x_{B,1})$$

But note that agent A's wealth hasn't increased, so this would mean that

$$x_{A,1}(\mathbf{p}, w_A \,|\, x^*_{B,1})p_1 + x_{A,2}(\mathbf{p}, w_A)p_2 > x_{A,1}(\mathbf{p}, w_A \,|\, x_{B,1})p_1 + x_{A,2}(\mathbf{p}, w_A)p_2.$$

However, the right-hand side of this inequality is equal to w_A by the assumption of Walras's Law that agent A was using her entire income before agent B

increased her consumption of good 1. But as we can see, the inequality now is breaking Walras's Law. Thus the amount of good 2 purchased by agent A must decrease. But this means that agent B's consumption of good 1 exhibits a negative celebrity effect on the consumption of good 2 by agent A. This could be called a "cross-commodity celebrity effect."

Note that the extension of this idea to more than two goods would require that the consumption of **at least one** of the other goods decrease, but that the "cross-commodity celebrity effect" could be spread across multiple goods. $\square$

Conclusion

I have shown a way of describing the consumption decisions when people have sway on others. This analysis can be extended to the objective functions maximized by the agents in order to derive the demand curves described above. We have shown, however, an important relationship that must hold in order to think about these types of demand curves.

References

[1] H. Varian, Microeconomic Analysis, Norton (1992).