Correction of Exploring Properties of Cayley Graphs of $\mathbb{Z}$ with Infinite Generating Sets

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We correct the proof of Theorem 6 in [1], using all the same notations and references given in [1].

Theorem 6. *Let $f : (\mathbb{Z}, d_2) \to (\mathbb{Z}, d_3)$ be a map defined with*

$$f(a) = \sum_{i=0}^{\infty} \varepsilon_i 3^i,$$

where $a = \sum_{i=0}^{\infty} \varepsilon_i 2^i \in \mathbb{Z}$, satisfying the conditions of [14, Theorem 3].

Then f is not a quasi-isometry.

Proof. Assume that f, as defined above, is a quasi-isometry.
Then there exist constants $k \geq 1$ and $c \geq 0$, such that

$$\frac{1}{k} d_2(a, b) - c \leq d_3(f(a), f(b)) \leq k d_2(a, b) + c.$$

Choose $n \in \mathbb{N}$ such that $2n > 2k + c$. Then take $a = 1 + 2^2 + \cdots + 2^{2n-2}$ and $b = -2a$. By Nathanson's length formula for C_2, it follows that

$$d_2(a, b) = l_2(a - b) = l_2(2^{2n} - 1) = 2.$$

In addition, by Nathanson's length formula for C_3, it follows that

$$d_3(f(a), f(b)) = l_3(f(a) - f(b)) = l_3(1 + 3 + 3^2 + \cdots + 3^{2n-1}) = 2n.$$

In this case,

$$d_3(f(a), f(b)) = 2n > 2k + c = k d_2(a, b) + c,$$

which contradicts the assumption that f is a quasi-isometry. □

References

[1] D. Adams, D. Gulbrandsen, V. Vasilevska, Exploring properties of Cayley graphs of the Integers with infinite generating sets, *Mathematics Exchange*, Vol. 10 No.1, (2016), 40-50.