A Word from the Editor

The editorial board is pleased to present our latest issue of the *Mathematics Exchange*, a collection of six articles of interest to a broad audience at the undergraduate level. We appreciate how authors inspire and motivate our readership to follow their example in sharing their love of mathematics, and we hope you enjoy the fruits of their labor. We believe that getting students involved in publishing mathematics is a true milestone in helping them find their (permanent) place in the mathematical community, and we are honored and proud to be a part of that endeavor.

Graph coloring is an important subfield of graph theory. The smallest number of colors needed to color the vertices of a graph so that no two adjacent vertices share the same color is referred to as the chromatic number of the graph. The first article reviews the history of 4-chromatic unit-distance graph, and extends the ideas of the Fish Graph (O'Donnell and Hochberg, 1996) to construct a 4-chromatic unit-distance graph containing only 21 vertices, which is an improvement over a construction of Hochberg and O'Donnell's graph of the same type of 23 vertices.

The second article is a very clear, understandable, and well-written expository paper on the famous *Čebotarev* Density Theorem. It provides all the details needed to prove the density theorem, and lists important applications showing that the density theorem has significant implications for primes in arithmetic progressions and binary quadratic forms. In addition, the explanations of difficult number theory concepts contained in this article, including L-series and density statements using Galois theory, are masterfully written. Reading this article will be enjoyable for both advanced undergraduate students and experts on *Čebotarev* densities.

The abundancy index of a positive integer is the ratio of the sum of its divisors and itself. The third article is an interesting expository article on the abundancy index. This is an accessible topic in elementary number theory, and it has some surprising connection to the Riemann hypothesis.

John H. Conway's Base-13 function is a nowhere-continuous, real-valued function on $\mathbb{R}$. It provides a counterexample to the converse of Intermediate Value Theorem on any interval of finite length. The value of the Base 13-function $f(x)$ can be described by manipulating the digits of x' s base-13 expansion. Although $f(x)$ can be easily defined in plain language, it is not trivial to formulate it in arithmetic alone. In the fourth article, the author constructs a closed-form function comprising only of arithmetic and proves that it is equivalent to the Base-13 function on integers.

The fifth article presents the application of mathematical modeling in financial problems. To study the optimal balance between savings and consumption, the authors apply Dynamic Programming and Optimal Control Theory in optimization models.

With the numerical simulation using the past data, they create an optimal monthly savings and consumption plan for the given financial goal.

The final article is another article dealing with an accessible topic in elementary number theory. It introduces a generalized definition of amicable numbers, discusses some related questions, and shows that some integers are not feebly amicable with any other integer. This article provides a good example of how an undergraduate research project can be devised.

We hope that you will enjoy reading this issue of the *Mathematics Exchange*. As always, we welcome and encourage ideas on how we can better serve our readers.

Yayuan Xiao

10.30.2021