A Word from the Editor

The editorial board of the *Mathematics Exchange* is pleased to present this latest issue, a collection of nine enjoyable articles addressing a range of mathematical topics of interest to a broad audience at the undergraduate level. We appreciate the authors' efforts to disseminate their new discoveries as well as appreciate how they inspire and motivate our readership to follow their example in sharing their love for mathematics. And we hope you enjoy the fruits of their labor. We believe that getting students involved in publishing mathematics is a true milestone in helping them find their (permanent) place in the mathematical community and we are thrilled to be a part of that endeavor.

The first article gives a detailed treatment of generalized trigonometric functions defined on the unit $p-$ circle $|x|^p + |y|^p = 1$. It covers the existing foundational material well, leads ideas in some interesting directions, provides a variety of new results, and lists some open questions for future work.

The second article considers quantities associated to a graph G called the "$k-$diameter component vertex connectivity parameter" and "$k-$diameter component connectivity function", denoted respectively as $CV_k(G)$ and $CM_k(G;p)$, and computes their values for several families of graphs. The quantity $CV_k(G)$ is the minimal number of vertices that needs to be removed from G so that no component of the remaining graph has diameter $\geq k$. The mixed parameter $CM_k(G;p)$ is similar but considers the minimal number of edges that need to be removed allowing for any p vertices to first be removed from G. The article is well written, and it should be quite accessible to undergraduates with some modest exposure to graph theory.

The third article provides a good example of mathematical modeling. It takes a standard stability analysis of an ordinary differential equations model approach to determine if a zombie apolocalypse is theoretically possible with realistic parameter values taken from empirical sources. The mathematics it presents is appropriate and reasonable and the application is fun.

It is well known that the sum of the entries along a slope-2 diagonal through Pascal's triangle is a Fibonacci number. For integers $h \leq 2$, the fourth article considers the sequence $d_h(n)$ of sums along slope-h diagonals, derives a recurrence and generating function for $d_h(n)$, and uses the generating function to obtain an approximation to $d_h(n)$.

A known characterization for entire functions that preserve all nonnegative matrices of order two is shown to characterize polynomials that preserve nonnegative matrices of order two. The fifth article gives a new characterization for polynomials that preserve nonnegative circulant matrices of order two.

Is it possible to dilate (or shrink) each side of a polygon $P \subset \mathbb{R}^2$ by a factor of positive t to get a new polygon P' while preserving the unit normal vectors to the edges? The sixth article draws a connection with Viviani's theorem and equiangular polygons, and uses the Minkowski's existence and uniqueness theorem for polytopes to show that such P' exists if and only if P satisfies the constant Viviani sum.

The classical Lotka-Volterra equation models the interaction between two species competing for limited resources; the seventh article explores its extension in which a general nonlinear relationship models the effects of each species on the other.

The eighth article gives a brief exposition of two different well-known metrics on the Heisenberg group, an extremely well-studied object in analysis, proves that there exist minimal geodesics for the Koranyi metric as a consequence of the Arzelà-Ascoli theorem, and shows that lengths of (horizontal) curves are the same when computed in either metric.

The final article investigates the numerical range of 3×3 matrices over finite fields, particularly when the matrix is strictly triangular. The article is both novel and deep. The reviewer rated this article highly, believing that it will have an impact on the field of research and be cited by future publications.

We hope that you will enjoy reading this issue of the *Mathematics Exchange*. As always, we welcome and encourage ideas on how we can better serve our readers.

Yayuan Xiao

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