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Exploring The Cosmic Wimpout Dice Game: Probabilities and a Markov Chain Model

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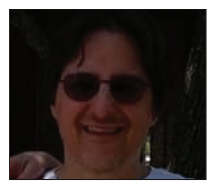

1 Introduction

Dice games, such as craps and Yahtzee, provide an opportunity for students to explore many mathematical concepts related to probability and modeling. One such dice game, Cosmic Wimpout, originated in the 1970s and is the focus of this article. Cosmic Wimpout is a game played with 5 special dice. Similar to craps, the rules governing Cosmic Wimpout force players to continue re-rolling 1, 2 or even all 5 dice at different times until certain results occur. In this article we investigate some probabilities associated with the Cosmic Wimpout dice game, develop a Markov Chain model for the game and apply matrix operations using the program Maple to predict the probable outcomes of the model.

2 Scoring and the Basic Rules of the Game

Cosmic Wimpout is a dice game played with four white dice with faces labeled 2, 3, 4, 5, 6, and 10, and one black die with a wild face referred to as the **Flaming**

Sun, *, replacing the 3. The rules for the game, which are explained below, can be found in various places, including the rules insert that accompanies the game and the official Cosmic Wimpout website [1]. The 5, 10 and Flaming Sun are considered scoring faces. The game can be played with any number of people, and the objective is to be the player with the most points at the end of the game. Players start a turn by rolling all 5 dice.

The following are the possible outcomes of the initial roll:

- For every 5 and 10 rolled, the player earns 5 and 10 points respectively, unless three or five of them are rolled at once. For example, if a player rolls 10 10 5 2 3, then the player scores $10 + 10 + 5 = 25$ points.
- If three faces match on a single roll, then the roll is scored as a Flash. Note the faces can have any value. The value of the matching faces is multiplied by 10 and scored. For example, if a player rolls 2 2 2 4 6, then the player scores 20 points.
- The Flaming Sun face is considered to be wild. It can be counted as either a 5 or a 10, or considered non-scoring and re-rolled except in the following instances.
  a. The Flaming Sun Rule – if the Flaming Sun is thrown in the same roll as a pair, then the Flaming Sun must be treated as the same value of the pair to make a Flash. For example, if a player rolls 4 4 * 2 6, then the Flaming Sun must be treated as a 4, and the player has a Flash of 4s worth 40 points. If the Flaming Sun is rolled with 2 different pairs, the player chooses which Flash to make.
  b. If the Flaming Sun is the only non-scoring die on a roll, then it must be counted as either a 5 or a 10 and set aside to avoid wimping out.
- If all five dice match, the roll is called a Freight Train, which scores as 100 times the value of the face. For example, if a player rolls 4 4 4 4 4, then the player scores 400 points. If a player rolls all sixes then the player instantly wins the game. However, if all tens are rolled, that is considered "too many points" and the player instantly loses the game. This is called a Supernova. Note if four faces match on a single roll and the other face is the Flaming Sun, this is NOT considered a Freight Train. For example, a roll of 4 4 4 4 * is treated as a Flash of 4s worth 40 points and the Flaming Sun can be valued as a 5, 10 or non-scoring.
- If the player manages to not roll a Freight Train, Flash, 5, 10 or the Flaming Sun, this is called a Wimpout (or wimping out) and the player receives no points.

Rules of the Game

If the player rolls and does not Wimpout, he or she has the option, or might even be forced to reroll a certain number of dice based on the following rules:

- The You May Not Want To But You Must (YMNWTBYM) rule forces a player to reroll all five dice if the player has scored on all five dice.
- The Futtless Rule forces a player to "clear" a Flash by rerolling all non-scoring dice. In order to clear a Flash, the player must roll any face not matching the Flash and score after rerolling all initial non-scoring dice. If a player rolls the same value as the Flash, then the player must reroll all dice that were not in the original Flash. If a player does not score on the reroll, he or she has wimped out and gets no points for the turn.
- The Flaming Sun Rule actually consists of two parts. If a pair is rolled along with the wild Flaming Sun face *, then the wild must take on the value of the pair to create a Flash. Secondly, if the Flaming Sun is the only scoring die rolled, it must take on a 5 or a 10 to prevent the player from wimping out. Typically, a player chooses the value to be 10 to maximize his or her score.
- The Opening Roll rule states that all players must score at least 35 points on his or her initial turn to be in the game. This means that a player cannot score his or her first points with any value below 35.

Outside of these rules, the player has discretion to decide whether or not he or she would like to reroll the non-scoring dice, and thus the math comes into play. Based on probabilities, strategies can be developed to ensure a higher success rate in not wimping out, as well as scoring more points. Once a player reaches the agreed winning total, a round of Last Licks begins. This round gives each player a last opportunity to score enough points to have a higher score than the first player to reach the winning value.

Deciphering Scores and Basic Strategies

For each of the example rolls, the highlighted value represents the black die.

Example Roll 1: 3 3 3 5 6

The Flash of 3s scores 30, and the 5 scores a 5, so the player scores 35. According to the Futtless Rule, he or she must reroll the 6 to clear the Flash of 3s. Recall the black die does not have a 3 on any of its faces. If a 5, 10, or the Flaming Sun is rolled (since the 6 was on the black die), the Flash is cleared and the last die's value would be added to the player's score, then the player must reroll all five dice due to the YMNWTBYM rule. However, if the player rolls a 2, 4, or 6 then he or she wimps out and scores no points.

Example Roll 2: 4 4 * 2 6

According to the Flaming Sun Rule, the wild must count as a 4, creating a Flash of 4s which is worth 40 points. By the Futtless Rule, the player must reroll the 2 and 6 to clear the Flash. If a 4 is rolled on either die, then by the Futtless Rule, the player must reroll both dice. If one of the faces is a 5 or 10 and the other is not a 4, the Flash is cleared and the player has the option to reroll the remaining die. At this point, the player could risk wimping out (by not scoring on the last die) in an attempt to increase his or her score. If the player rerolls scoring values on both remaining dice, then by the YMNWTBYM rule, the player must continue his or her turn by rerolling all five dice.

Example Roll 3: 2 2 2 2 10

On this roll, a Flash of 2s is scored, but only using three of the four 2s rolled. The Flash along with the additional 10 values this roll at 30. According to the Futtless Rule, one of the 2s must be rerolled in order to clear the Flash of 2s. The player then decides which die to reroll. Statistically speaking, the player has a higher chance of wimping out if a white die is rerolled, since only two faces are scoring, whereas the black die has three scoring faces. Therefore the assumed strategy is to reroll the black die in an attempt to score and avoid wimping out.

Example Roll 4: 3 5 3 10 3

The Flash of 3s scores 30, then the 5 and 10 score a total of 15, so this roll's value is 45. According to the YMNWTBYM rule, the player must reroll all five dice. On the reroll, the player still must clear the Flash of 3s due to the Futtless rule. If the player wimps out on the next roll, however, then he or she would receive no points for this turn.

Example Roll 5: 5 10 2 * 3

The 5 and 10 score a total of 15, and to obtain the most points, the Flaming Sun would represent 10, valuing this roll at 25. As long as this roll is not the initial roll, which by the Opening Roll rule requires 35 points to get in the game, then the player has the option to either stop his or her turn and take the 25 points, or risk wimping out by rolling the two non-scoring dice

Example Roll 6: 2 2 2 2 *

The player rolled four 2s and the Flaming Sun, but the Sun does not count as a 2 to create a Freight Train. Instead, the player would score 20 from a Flash of 2s, and an additional 10 from the Flaming Sun. Then the player must clear the Flash by rerolling one of the 2s and rolling either a 5 or 10, forcing a reroll of all five dice by the YMNWTBYM rule. If the player's reroll results with a 2 again, then he or she must continue rerolling, but if he or she rolls a 3, 4, or 6, then the player wimps out.

3 Game Scores for Cosmic Wimpout

To calculate probabilities of particular rolls and scores, we must first know all possible outcomes of the initial roll. Creating a full list of all possible outcomes of the initial roll resulted in 25 possible numerical outcomes along with the non-numerical outcomes of automatically winning or automatically losing the game. For each possible numerical outcome, another comprehensive list was made showing each possible way to score that particular number of points on the initial roll. Using permutations, combinations and basic counting techniques as described in [3], we can determine through brute force how many unique ways that score can occur.

For a detailed example of this, we will consider the score value of 40. Table 1 lists all possible ways to score 40 points, organized by whether or not the Flaming Sun was rolled, and what Flash value was rolled. Here a and b are distinct and represent a variable non-scoring face, which is either a 2, 3, 4, or 6 on a white die and either a 2, 4, or 6 on a black die. Another important point is if the row contains non-scoring variables, the variable die or dice cannot represent the scoring value. For instance, in the roll $\underline{4}\ \underline{4}\ \underline{4}\ \underline{a}\ \underline{b}$, neither the a nor the b can be a 4. The final column in the table displays the mathematical representations of the number of permutations of each row.

Considering the roll $\underline{4}\ \underline{4}\ \underline{4}\ \underline{4}\ \colorbox{lightgray}{$\underline{a}$}$, we see that all white dice are 4 and the black die is a non-scoring face, which is either a 2 or a 6 in this case. We find the permutations of the white dice by $\frac{4!}{4!} = 1$. We then have to determine how many combinations we have for the black die. We find that the count is $\binom{2}{1} = 2$ since only two possible faces can be rolled, and we have to choose one of the two. Multiplying these values gives us 2 ways to score 40 points with the roll $\underline{4}\ \underline{4}\ \underline{4}\ \underline{4}\ \colorbox{lightgray}{$\underline{a}$}$.

Considering the roll $\underline{4}\ \underline{4}\ \underline{a}\ \underline{b}\ \colorbox{lightgray}{$\underline{4}$}$, we see that the two variable dice are different faces on white dice. Since these faces can only be 2, 3, or 6 and we need to select two of the three numbers, we have $\binom{3}{2} = 3$ different possible combinations for the unknown faces. Seeing that we have two 4s, and one of each variable die, we have $\frac{4!}{2!1!1!} = 12$ possible ways to arrange the white dice. Since the black die is fixed, we do not need to account for this separately. Multiplying these values gives us 36 ways to score 40 points with the roll $\underline{4}\ \underline{4}\ \underline{a}\ \underline{b}\ \colorbox{lightgray}{$\underline{4}$}$.

To consider an example with the Flaming Sun, we can look at the roll $\underline{3}\ \underline{3}\ \underline{10}\ \underline{a}\ \colorbox{lightgray}{$\underline{*}$}$. For this roll, the unknown white die can be either a 2, 4, or 6. Since we need one of the three possible outcomes, we use $\binom{3}{1} = 3$ to find the number of different combinations for the unknown die. Since we have four white dice, two of which are 3s, along with one 10 and one unknown, we have $\frac{4!}{2!1!1!} = 12$ permutations of the white dice. The Flaming Sun is a constant in this case, so we do not need to consider it separately. Multiplying these values gives us 36 ways to score 40 points with the roll $\underline{3}\ \underline{3}\ \underline{10}\ \underline{a}\ \colorbox{lightgray}{$\underline{*}$}$.

We also want to focus on a specific roll example that requires some care; the roll $\underline{4}\ \underline{4}\ \underline{a}\ \underline{a}\ \colorbox{lightgray}{$\underline{*}$}$. For this roll, the unknown white die can only be 2 or 3. It cannot be 6 because the roll $\underline{4}\ \underline{4}\ \underline{6}\ \underline{6}\ \underline{*}$ would be counted as the higher value of 60. Since we need one of only two possible outcomes, we use $\binom{2}{1} = 2$ to find the number of different combinations for the unknown die. Since we have 4 white dice, two of which are 4 and the other two are the same value a, we have $\frac{4!}{2!2!} = 6$ permutations of the white dice. Since the black die is fixed, we do not need to account for this separately. Multiplying these values gives us 12 distinct ways to score 40 with the roll $\underline{4}\ \underline{4}\ \underline{a}\ \underline{a}\ \underline{*}$.

Continuing these calculations for all possible rolls for this value, we find 232 different ways to score 40 points as shown in Table 1.

Table 1: All 232 total possible combinations of the score 40.

4	4	4	4	**a**	$\binom{2}{1}\frac{4!}{4!} = 2$
4	4	a	a	**4**	$\binom{3}{1}\frac{4!}{2!2!} = 18$
4	4	4	a	$\boldsymbol{a}$	$\binom{2}{1}\frac{4!}{3!\,1!} = 8$
4	4	a	b	**4**	$\binom{3}{2}\frac{4!}{2!\,1!\,1!} = 36$
4	4	4	a	$\boldsymbol{b}$	$\binom{2}{1}\binom{2}{1}\frac{4!}{3!\,1!} = 16$
4	4	4	a	**4**	$\binom{3}{1}\frac{4!}{3!\,1!} = 12$
3	3	3	a	**10**	$\binom{3}{1}\frac{4!}{3!\,1!} = 12$
3	3	3	10	$\boldsymbol{a}$	$\binom{3}{1}\frac{4!}{3!\,1!} = 12$
3	3	3	3	**10**	$\frac{4!}{4!} = 1$
2	2	2	10	**10**	$\frac{4!}{3!\,1!} = 4$
10	10	2	2	**2**	$\frac{4!}{2!2!} = 6$
3	3	3	5	**5**	$\frac{4!}{3!\,1!} = 4$
4	4	a	b	*	$\binom{3}{2}\frac{4!}{2!\,1!\,1!} = 36$
4	4	a	a	*	$\binom{2}{1}\frac{4!}{2!2!} = 12$
3	3	10	a	*	$\binom{3}{1}\frac{4!}{2!\,1!\,1!} = 36$
2	2	2	10	*	$\frac{4!}{3!\,1!} = 4$
3	3	3	a	*	$\binom{3}{1}\frac{4!}{3!\,1!} = 12$
3	3	3	3	*	$\frac{4!}{4!} = 1$

Remembering the grand total of 7,776 ways to roll five dice, we can expand on this list to create the frequency distribution, Table 2, which summarizes the number of ways to get each possible result of the initial roll. As mentioned previously, all numerical score values were determined via brute force using permutations, combinations and basic counting techniques.

Table 2: Possible initial roll value along with the associated frequency count.

Roll Value	Frequency
0	450
5	816
10	1350
15	1116
20	1166
25	673
30	315
35	97
40	232
45	121
50	362
55	55
60	300
65	103
70	140
75	36
80	14
100	240
105	124
110	51
115	9
120	1
200	1
400	1
500	1
Auto Win	1
Auto Lose	1
Total	**7776**

4 Expected Value and Variance of Cosmic Wimpout Initial Roll

The expected value of an experiment is the weighted average of all numerical outcomes, and this value predicts the average score for the player to roll on his or her initial roll. If we let X be the value of the initial roll, then using the data from Table 2, we can find the expected value, $E[X]$, and the variance, $Var[X]$, for a player's initial roll. For the purpose of these computations, we will exlcude the automatically winning and automatically losing rolls, as these are not actual numerically scored values possible to use in calculations and just use the other 7774 outcomes. Table 4 below was created using Excel and includes the probabilities of each roll value X along with the expected value and variance. From this we can expect, on average, any player in his or her initial roll to score 26.54 and we have a variance of about 690 points.

Table 3: The probabilities used to calculate E[X] and Var[X].

X	P(X)	X ∗ P(X)
0	0.057885259	0
5	0.104965269	0.5248263
10	0.173655776	1.7365577
15	0.143555441	2.1533316
20	0.149987137	2.9997427
25	0.08657062	2.1642655
30	0.040519681	1.2155904
35	0.012477489	0.4367121
40	0.029843067	1.1937226
45	0.015564703	0.7004116
50	0.046565475	2.3282737
55	0.007074865	0.3891175
60	0.038590172	2.3154103
65	0.013249293	0.8612040
70	0.018008747	1.2606122
75	0.004630821	0.3473115
80	0.001800875	0.1440699
100	0.030872138	3.0872137
105	0.015950605	1.6748134
110	0.006560329	0.7216362
115	0.001157705	0.1331360
120	0.000128634	0.0154360
200	0.000128634	0.0257267
400	0.000128634	0.0514535
500	0.000128634	0.0643169
	Expected Value:	**26.544893**
	Variance:	**690.31976**

5 Markov Chains Applied to Cosmic Wimpout

Following the terminology found in [2], we describe a Markov chain as a set of states along with a process that starts in one of the states and moves from one state to another one step at a time. The transitional probabilities represent the likelihood of flowing from one state to another, and the sum of all probabilities of leaving each individual state is 1. Some Markov chains consist of one or more states in which, once they are entered, they cannot be exited. In this case, the chain would be called an Absorbing Markov chain, and any state which cannot be exited is called an absorbing state. Numerically speaking, for any absorbing state the probability of transitioning to any other state is 0, while the probability of transitioning to itself is 1. In an Absorbing Markov chain, any state that is not absorbing is called a transient state. Once the process has reached an absorbing state, we call the process absorbed. Visually, small Markov chains can be represented by digraphs where the vertices (or circles) represent the states and the directed edges (or arrows) represent the transitional probabilities. However, when describing Markov Chains, it is often common to use matrices whose entries are based on the probabilities of transitioning from one state to another. Specifically, a transition matrix is one that incorporates the transitional probabilities of a Markov chain, allowing us to answer long-term questions about the process.

The Markov chain model we created for the Cosmic Wimpout Dice Game consists of 21 states. Without loss of generality, we can group the states based on the value of a Flash rolled (if any), as well as how many and what color die or dice required rerolling. We also can combine certain Flash values into a single state due to identical probabilities of leaving states. This method of organization results in a model with four absorbing states. The absorbing states are automatically winning, automatically losing, scoring and stopping, and wimping out. The 17 transient states are listed below in Table 4, where B represents a black die and W represents a white die. The final transient state listed, "Roll/Reroll 5 dice, No Flash to clear" represents any roll of the five dice with no Flash to clear, which is also representative of the initial roll.

Table 4: The 17 Transient states from the Cosmic Wimpout Markov Chain.

Flash 2, 3, 4 or 6; Reroll 1W
Flash 5 or 10; Reroll 1W
Flash, 2, 4 or 6; Reroll 1B
Flash 3; Reroll 1B
Flash 5 or 10; Reroll 1B
Flash 2, 3, 4 or 6; Reroll 2W
Flash 5 or 10; Reroll 2W
Flash 2, 4 or 6; Reroll 1B and 1W
Flash 3; Reroll 1B, 1W
Flash 5 or 10; Reroll 1B and 1 W
Flash 2; Reroll 5 dice
Flash 3; Reroll 5 dice
Flash 4; Reroll 5 dice
Flash 5; Reroll 5 dice
Flash 6; Reroll 5 dice
Flash 10; Reroll 5 dice
Roll/Reroll 5 dice; No Flash to clear

By adding certain reasonable assumptions regarding specific cases during game play we can minimize the number of variables and simplify the study significantly. Generally, we assume the player will always make rational decisions. The first assumption is that if a player rolls a Flaming Sun, when not counted as part of a Flash, the value will be 10 instead of 5 since that results in more total points. The second assumption is that a player will only reroll the die or dice if forced to do so by the rules of the game. Thirdly, we assume that if a player has to clear a Flash and has a choice to reroll a white or black die, he or she will choose to reroll the black die to maximize the chance of scoring and clearing the Flash. Operating under these assumptions, we can calculate transitional probabilities from each state and complete the values represented in the Markov model. We can then arrange these probabilities in a 21 x 21 transition matrix.

After determining the possible outcomes for the initial roll, the possible outcomes for all rerolls have to be accounted for in order to eventually create the Markov Chain model. For example, when clearing a Flash, depending on whether or not there are other scoring dice, there is the possibility of rerolling either one white die, one black die, one white and one black die, two white dice, or all 5 dice. Consider the example of having to reroll two dice to clear a Flash of 2s where one die is white and the other is black, which means we are in the state Flash **2, 4 or 6; Reroll 1B and 1W**. There are 36 possible outcomes when we reroll, but each results in transitioning to one of four states. These states are scoring and stopping, scoring and rerolling all 5 dice with no Flash to clear, wimping out, or getting a 2 on one of the rerolled dice and, by the Futtless Rule, remaining in the same state of **Flash 2, 4 or 6; Reroll 1B and 1W**. Note two of these states are transient and the other two are absorbing. A list of all possible reroll outcomes for this example along with their corresponding state transitioned to appears in Table 5.

Table 5: Outcomes For Rerolling One White Die and One Black Die For Initial Roll Flash of 2s.

2	2	Reroll same dice
2	*	Reroll same dice
2	4	Reroll same dice
2	5	Reroll same dice
2	6	Reroll same dice
2	10	Reroll same dice
3	2	Reroll same dice
3	*	Score/Stop
3	4	Wimpout
3	5	Score/Stop
3	6	Wimpout
3	10	Score/Stop
4	2	Reroll same dice
4	*	Score/Stop
4	4	Wimpout
4	5	Score/Stop
4	6	Wimpout
4	10	Score/Stop

5	2	Reroll same dice
5	*	Score/Reroll 5
5	4	Score/Stop
5	5	Score/Reroll 5
5	6	Score/Stop
5	10	Score/Reroll 5
6	2	Reroll same dice
6	*	Score/Stop
6	4	Wimpout
6	5	Score/Stop
6	6	Wimpout
6	10	Score/Stop
10	2	Reroll same dice
10	*	Score/Reroll 5
10	4	Score/Stop
10	5	Score/Reroll 5
10	6	Score/Stop
10	10	Score/Reroll 5

As we can see, 11 of the 36 outcomes result in the player rerolling the same two dice and staying in the same state, 13 result in the player scoring and stopping, 6 result in the player scoring on all five dice and being forced to reroll all five but no longer having to clear a Flash, and 6 result in wimping out. This basic approach was used to determine the transitional probabilities for the entire Markov chain model.

We now present some examples illustrating how to completely determine the transitional probabilities between states. For each example, we will not reduce the fractions in order to highlight the counting aspects involved. Note that going forward, the states will be abbreviated for space considerations. For example, **Flash 3** means we have a Flash of 3s, **Roll 5** means we must roll all 5 dice, **No Flash** means no Flash needs to be cleared, etc...

<u>Transitional Probability Example 1</u>: Suppose we are in the state **Flash 3; Reroll 1B**. There are only 2 states we can transition to:

- If we score on the reroll, we have cleared the Flash, but have scored on all 5 dice. Because of the YMNWTBYM rule, we must now reroll all 5 dice, but no longer have to clear a Flash. This means we have transitioned to the transient state **Roll 5; no Flash**. There are 3 values of the black die which give this result; namely 5, 10, and *. Thus, the transitional probability from **Flash 3; Reroll 1B** to **Roll 5; no Flash** is $\frac{3}{6}$.
- If we do not score on the reroll, this means we have rolled a 2, 4, or 6 (recall the black die does not have a value of 3). This means our turn ends and we have transitioned to the absorbing state **Wimpout**. There are 3 values of the black die which satisfy this condition: namely 2, 4, and 6. Thus, this transitional probability from **Flash 3; Reroll 1B** to **Wimpout** is $\frac{3}{6}$.
- Since we cannot transition from the state **Flash 3; Reroll 1B** back to itself or any of the other 18 states those transitional probabilities are all 0.

<u>Transitional Probability Example 2</u>: Suppose we are in the state **Flash 5, 10; Reroll 1W**. Without loss of generality, we will assume we have a Flash of 5s. We can transition to 3 possible states.

- If we get a 5 on the reroll, we have not cleared the Flash, and by the Futtless rule we must reroll the white die again. This means we have transitioned back to the same state of **Flash 5, 10; Reroll 1W**. Thus, the transitional probability from **Flash 5, 10; Reroll 1W** back to **Flash 5, 10; Reroll 1W** is $\frac{1}{6}$.
- If we get a 10 on the reroll, we have cleared the Flash, but have now scored on all 5 dice. Because of the YMNWTBYM rule, we must now reroll all 5 dice, but no longer have to clear a Flash. This means we have transitioned to the transient state **Roll 5; no Flash**. Thus, the transitional probability from **Flash 5, 10; Reroll 1W** to **Roll 5; no Flash** is $\frac{1}{6}$.

- If we get a 2, 3, 4, or 6 on the reroll, then our turn ends and we transition to the absorbing state **Wimpout**. Thus, the transitional probability from **Flash 5, 10; Reroll 1W** to **Wimpout** is $\frac{4}{6}$.

- Since we cannot transition from the state **Flash 5, 10; Reroll 1W** to any of the remaining 18 states, those transitional probabilities are all 0.

<u>Transition Probability Example 3</u>: Suppose we are in the state **Flash 2, 4, 6; Reroll 1B, 1W**. Without loss of generality, assume that we have a Flash of 2s. Table 5 actually outlines all the possible states we can transition to in this situation.

- If we reroll and either of the two dice rolls a 2, then we have not cleared the Flash and by the Futtless rule, we must reroll both dice again. This means we have transitioned back to the transient state **Flash 2, 4, 6; Reroll 1B, 1W**. As seen in Table 5, there are 11 ways this can occur. Thus, the transitional probability from **Flash 2, 4, 6; Reroll 1B, 1W** back to **Flash 2, 4, 6; Reroll 1B, 1W** is $\frac{11}{36}$.

- If we reroll and both dice end up as a scoring die (so the white die is 5 or 10 and the black die is 5, 10, or *), then we have cleared the Flash, but have now scored on all 5 dice. Because of the YMNWTBYM rule, we must now reroll all 5 dice, but no longer have to clear a Flash. This means we have transitioned to the transient state **Roll 5; no Flash**. As seen in Table 5, there are 6 ways this can occur. Thus, the transitional probability from **Flash 2, 4, 6; Reroll 1B, 1W** to **Roll 5; no Flash** is $\frac{6}{36}$.

- If we reroll and score on exactly one of the dice, and the value on the other die is not a 2, then we have cleared the Flash, and since we did not score on all 5 dice we can actually score points and stop our turn. This means we have transitioned to the absorbing state **Score and Stop**. As seen in Table 5, there are 13 ways this can occur. Thus, the transitional probability from **Flash 2, 4, 6; Reroll 1B, 1W** to **Score and Stop** is $\frac{13}{36}$.

- If we reroll and neither die ends up a scoring die and, also, neither die rolls a 2, then our turn ends and we transition to the absorbing state **Wimpout**. As seen in Table 5, there are 6 ways this can occur. Thus, the transitional probability from **Flash 2, 4, 6; Reroll 1B, 1W** to **Wimpout** is $\frac{6}{36}$.

- Since we cannot transition from the state **Flash 2, 4, 6; Reroll 1B, 1W** to any of the remaining 17 states, those transitional probabilities are all 0.

This process can be done for all 21 states in order to compute all transitional probabilities for the Markov Chain and then be used to create a transition matrix. Following the notation in [2], we will arrange the matrix, which we will denote *P,* in a particular form called canonical form. This entails listing all transient states first, followed by all absorbing states. Once the Cosmic Wimpout Markov chain model is in canonical form, we can see the pattern, $P = \begin{pmatrix} Q & R \\ 0 & I \end{pmatrix}$. Here 0 represents a 4 x 17 matrix in which every entry is 0, I represents a 4 x 4 identity matrix, Q is the 17 x 17 matrix consisting of the probabilities of transitioning from transient states to transient states, and R is the 17 x 4 matrix consisting of the probabilities of transitioning from transient states to absorbing states. The entire matrix P, shown in Table 6 (which has been broken up into four smaller tables so all values can be read more easily), can now be operated on using Maple software to answer questions regarding the long-term process of the game.

Table 6: Tables 6a-6d together show the entries of matrix P.

Table 6a: Transition matrix representing the Markov Chain Model for Cosmic Wimpout. All Initial States with Ending States of Rerolling 1W or 1B.

	Flash 2, 3, 4, 6; Reroll 1W	**Flash 5, 10; Reroll 1W**	**Flash 2 ,4, 6; Reroll 1B**	**Flash 3; Reroll 1B**	**Flash 5, 10; Reroll 1B**
Flash 2, 3, 4, 6; Reroll 1W	1/6	0	0	0	0
Flash 5, 10; Reroll 1W	0	1/6	0	0	0
Flash 2, 4, 6; Reroll 1B	0	0	1/6	0	0
Flash 3; Reroll 1B	0	0	0	0	0
Flash 5, 10; Reroll 1B	0	0	0	0	1/6
Flash 2, 3, 4, 6; Reroll 2W	0	0	0	0	0
Flash 5, 10 ; Reroll 2W	0	0	0	0	0
Flash 2, 4, 6; Reroll 1B, 1W	0	0	0	0	0
Flash 3; Reroll 1B, 1W	0	0	0	0	0
Flash 5, 10; Reroll 1B, 1W	0	0	0	0	0
Flash 2; Reroll 5	321/7776	216/7776	32/77776	16/7776	20/7776
Flash 3; Reroll 5	369/7776	216/7776	72/7776	0	30/7776
Flash 4; Reroll 5	321/7776	216/7776	32/7776	16/7776	20/7776
Flash 5; Reroll 5	356/7776	32/7776	36/7776	12/7776	3/7776
Flash 6; Reroll 5	321/7776	216/7776	32/7776	16/7776	20/7776
Flash 10; Reroll 5	356/7776	32/7776	36/7776	12/7776	3/7776
Roll 5; No Flash	660/7776	288/7776	72/7776	24/7776	30/7776
Auto Win	0	0	0	0	0
Auto Lose	0	0	0	0	0
Score and Stop	0	0	0	0	0
Wimpout	0	0	0	0	0

Table 6b: Transition matrix representing the Markov Chain Model for Cosmic Wimpout. All Initial States with Ending States of Rerolling 2W or 1B,1W.

	Flash 2, 3, 4, 6; Reroll 2W	Flash 5,10 ; Reroll 2W	Flash 2, 4, 6; ReRoll 1B, 1W	Flash 3; Reroll 1B, 1W	Flash 5,10; Reroll 1B, 1W
Flash 2, 3, 4, 6; Reroll 1W	0	0	0	0	0
Flash 5, 10; Reroll 1W	0	0	0	0	0
Flash 2, 4, 6; Reroll 1B	0	0	0	0	0
Flash 3; Reroll 1B	0	0	0	0	0
Flash 5, 10; Reroll 1B	0	0	0	0	0
Flash 2, 3, 4, 6; Reroll 2W	11/36	0	0	0	0
Flash 5, 10 ; Reroll 2W	0	11/36	0	0	0
Flash 2, 4, 6; Reroll 1B, 1W	0	0	11/36	0	0
Flash 3; Reroll 1B, 1W	0	0	0	6/36	0
Flash 5, 10; Reroll 1B, 1W	0	0	0	0	11/36
Flash 2; Reroll 5	102/7776	210/7776	34/7776	18/7776	48/7776
Flash 3; Reroll 5	126/7776	210/7776	78/7776	0	72/7776
Flash 4; Reroll 5	102/7776	210/7776	34/7776	18/7776	48/7776
Flash 5; Reroll 5	342/7776	192/7776	114/7776	39/7776	48/7776
Flash 6; Reroll 5	102/7776	216/7776	34/7776	18/7776	48/7776
Flash 10; Reroll 5	342/7776	186/7776	114/7776	39/7776	48/7776
Roll 5; No Flash	342/7776	378/7776	114/7776	39/7776	96/7776
Auto Win	0	0	0	0	0
Auto Lose	0	0	0	0	0
Score and Stop	0	0	0	0	0
Wimpout	0	0	0	0	0

Table 6c: Transition matrix representing the Markov Chain Model for Cosmic Wimpout. All Transient States with Ending States of Rerolling or Rolling all 5 Dice.

	Flash 2; Reroll 5	Flash 3; Reroll 5	Flash 4; Reroll 5	Flash 5; Reroll 5	Flash 6; Reroll 5	Flash 10; Reroll 5	Roll 5; No Flash
Flash 2, 3, 4, 6; Reroll 1W	0	0	0	0	0	0	2/6
Flash 5, 10; Reroll 1W	0	0	0	0	0	0	1/6
Flash 2, 4, 6; Reroll 1B	0	0	0	0	0	0	3/6
Flash 3; Reroll 1B	0	0	0	0	0	0	3/6
Flash 5, 10; Reroll 1B	0	0	0	0	0	0	2/6
Flash 2, 3, 4, 6; Reroll 2W	0	0	0	0	0	0	4/36
Flash 5, 10; Reroll 2W	0	0	0	0	0	0	1/36
Flash 2, 4, 6; Reroll 1B, 1W	0	0	0	0	0	0	6/36
Flash 3; Reroll 1B, 1W	0	0	0	0	0	0	6/36
Flash 5, 10; Reroll 1B, 1W	0	0	0	0	0	0	2/36
Flash 2; Reroll 5	4651/7776	36/7776	60/7776	20/7776	66/7776	26/7776	2/7776
Flash 3; Reroll 5	60/7776	4026/7776	60/7776	20/7776	66/7776	26/7776	3/7776
Flash 4; Reroll 5	60/7776	36/7776	4651/7776	20/7776	66/7776	26/7776	2/7776
Flash 5; Reroll 5	14/7776	8/7776	14/7776	4651/7776	14/7776	1/7776	2/7776
Flash 6; Reroll 5	60/7776	36/7776	60/7776	20/7776	4651/7776	26/7776	3/7776
Flash 10; Reroll 5	14/7776	8/7776	14/7776	1/7776	20/7776	4651/7776	3/7776
Roll 5; No Flash	60/7776	36/7776	60/7776	20/7776	66/7776	26/7776	3/7776
Auto Win	0	0	0	0	0	0	0
Auto Lose	0	0	0	0	0	0	0
Score and Stop	0	0	0	0	0	0	0
Wimpout	0	0	0	0	0	0	0

Table 6d: Transition matrix representing the Markov Chain Model for Cosmic Wimpout. All Initial States with Ending States an Absorbing State.

	Auto Win	**Auto Lose**	**Score and Stop**	**Wimpout**
Flash 2, 3, 4, 6; Reroll 1W	0	0	0	3/6
Flash 5, 10; Reroll 1W	0	0	0	4/6
Flash 2, 4, 6; Reroll 1B	0	0	0	2/6
Flash 3; Reroll 1B	0	0	0	3/6
Flash 5, 10; Reroll 1B	0	0	0	3/6
Flash 2, 3, 4, 6; Reroll 2W	0	0	12/36	9/36
Flash 5, 10 ; Reroll 2W	0	0	8/36	16/36
Flash 2, 4, 6; Reroll 1B, 1W	0	0	13/36	6/36
Flash 3; Reroll 1B, 1W	0	0	15/36	9/36
Flash 5, 10; Reroll 1B, 1W	0	0	11/36	12/36
Flash 2; Reroll 5	1/7776	1/7776	1836/7776	60/7776
Flash 3; Reroll 5	1/7776	1/7776	2250/7776	90/7776
Flash 4; Reroll 5	1/7776	1/7776	1836/7776	60/7776
Flash 5; Reroll 5	1/7776	1/7776	1446/7776	450/7776
Flash 6; Reroll 5	0	1/7776	1836/7776	60/7776
Flash 10; Reroll 5	1/7776	0	1446/7776	450/7776
Roll 5; No Flash	1/7776	1/7776	5010/7776	450/7776
Auto Win	1	0	0	0
Auto Lose	0	1	0	0
Score and Stop	0	0	1	0
Wimpout	0	0	0	1

Expected Number of Times in Transient States

As described in [2], the first matrix operation to perform gives us what we call the fundamental matrix, N. We find N by the formula $N = (I - Q)^{-1}$ where I is a 17 x 17 identity matrix and Q we find from the canonical form of P. We call the states in each row s_i and the states in each column s_j. These matrix entries represent the expected number of times the process will be in each transient state s_j over a long period of time, given the process started in state s_i. The matrix N for our model was computed using Maple and is given in Table 7 below:

Table 7: Fundamental Matrix N for Cosmic Wimpout Markov Chain.

1.246	0.02015	0.004988	0.001390	0.002093	0.02822	0.03151	0.009436	0.002698	0.007987	0.008705	0.004393	0.008705	0.002948	0.009561	0.003832	0.4291
0.02283	1.210	0.002494	0.0006952	0.001047	0.01411	0.01575	0.004718	0.001349	0.003994	0.004352	0.002196	0.004352	0.001474	0.004781	0.001916	0.2146
0.06850	0.03023	1.207	0.002086	0.003140	0.04233	0.04726	0.01415	0.004048	0.01198	0.01306	0.006589	0.01306	0.004422	0.01434	0.005748	0.6437
0.05708	0.02519	0.006235	1.002	0.002617	0.03527	0.03938	0.01179	0.003373	0.009984	0.01088	0.005491	0.01088	0.003685	0.01195	0.004790	0.5364
0.04567	0.02015	0.004988	0.001390	1.202	0.02822	0.03151	0.009436	0.002698	0.007987	0.008705	0.004393	0.008705	0.002948	0.009561	0.003832	0.4291
0.01827	0.008060	0.001995	0.0005562	0.0008374	1.451	0.01260	0.003774	0.001079	0.003195	0.003482	0.001757	0.003482	0.001179	0.003824	0.001533	0.1716
0.004567	0.002015	0.0004988	0.0001390	0.0002093	0.002822	1.443	0.0009436	0.0002698	0.0007987	0.0008705	0.0004393	0.0008705	0.0002948	0.0009561	0.0003832	0.04291
0.02740	0.01209	0.002993	0.0008342	0.001256	0.01693	0.01890	1.446	0.001619	0.004792	0.005223	0.002636	0.005223	0.001769	0.005737	0.002299	0.2575
0.02283	0.01008	0.002494	0.0006952	0.001047	0.01411	0.01575	0.004718	1.201	0.003994	0.004352	0.002196	0.004352	0.001474	0.004781	0.001916	0.2146
0.009133	0.004030	0.0009975	0.0002781	0.0004187	0.005644	0.006301	0.001887	0.0005397	1.442	0.001741	0.0008786	0.001741	0.0005897	0.001912	0.0007664	0.08582
0.1418	0.09183	0.01437	0.005696	0.008590	0.05807	0.1100	0.01953	0.008017	0.02545	2.493	0.02593	0.05138	0.01740	0.05644	0.02262	0.09173
0.1361	0.07795	0.02494	0.0006143	0.01047	0.05897	0.09366	0.03350	0.001133	0.03082	0.04353	2.076	0.04353	0.01474	0.04782	0.01916	0.09217
0.1418	0.09183	0.01437	0.005696	0.008590	0.05807	0.1100	0.01953	0.008017	0.02545	0.05138	0.02593	2.493	0.01740	0.05644	0.02262	0.09173
0.1493	0.01826	0.01521	0.004232	0.001765	0.1649	0.09732	0.05502	0.01570	0.02434	0.01370	0.006652	0.01370	2.489	0.01395	0.002011	0.09797
0.1416	0.09167	0.01435	0.005687	0.008576	0.05799	0.1126	0.01950	0.008005	0.02541	0.05130	0.02589	0.05130	0.01737	2.493	0.02258	0.09198
0.1496	0.01845	0.01524	0.004244	0.001783	0.1650	0.09479	0.05506	0.01572	0.02440	0.01380	0.006705	0.01380	0.001766	0.01874	2.490	0.09841
0.1142	0.05038	0.01247	0.003476	0.005233	0.07055	0.07876	0.02359	0.006746	0.01997	0.02176	0.01098	0.02176	0.007371	0.02390	0.009580	1.073

Absorbing Probabilities

As noted in [2], raising P to any power n results with a 21 x 21 matrix where the i,j entry represents the probability of being in state s_j after n steps, given the player started in state s_i. Therefore, as n approaches infinity, the entries in block Q^n all approach 0. The entries in the upper right block represent the probabilities that the chain will be absorbed into state s_j given that the player started in transient state s_i. Using R from the canonical form of matrix P, we can compute the product $B = NR$, where N is the fundamental matrix found previously. The entries in matrix B represent the probability that the chain will be absorbed into state s_j given that the process started in transient state s_i. The matrix B for our model was computed using Maple software and is given below in Table 8. Since the last row also represents the initial state of the game, we know that the probabilities in this row denote the probabilities that a player will end in each respective absorbing state by following the assumptions outlined previously for game play.

Table 8: Matrix of Absorbing Probabilities For Each Transient State.

	Auto Win	Auto Lose	Score & Stop	Wimpout
Flash 2, 3, 4, 6; Reroll 1W	0.00005886112400	0.00005959788628	0.3087586108	0.6911229302
Flash 5, 10; Reroll 1W	0.00002943056200	0.00002979894314	0.1543793054	0.8455614651
Flash 2, 4, 6; Reroll 1B	0.00008829168600	0.00008939682941	0.4631379162	0.5366843953
Flash 3; Reroll 1B	0.00007357640500	0.00007449735784	0.3859482635	0.6139036628
Flash 5, 10; Reroll 1B	0.00005886112400	0.00005959788628	0.3087586108	0.6911229302
Flash 2, 3, 4, 6; Reroll 2W	0.00002354444960	0.00002383915451	0.6035034443	0.3964491721
Flash 5, 10; Reroll 2W	0.000005886112400	0.000005959788628	0.3508758611	0.6491122930
Flash 2, 4, 6; Reroll 1B, 1W	0.00003531667440	0.00003575873177	0.7052551665	0.2946737581
Flash 3; Reroll 1B, 1W	0.00002943056200	0.00002979894314	0.6543793054	0.3455614651
Flash 5, 10; Reroll 1B, 1W	0.00001177222480	0.00001191957726	0.5017517222	0.4982245860
Flash 2; Reroll 5	0.0003474672870	0.0003518165208	0.7500709250	0.2492297912
Flash 3; Reroll 5	0.0002943662535	0.0002980508238	0.7606404280	0.2387671549
Flash 4; Reroll 5	0.0003474672870	0.0003518165208	0.7500709250	0.2492297912
Flash 5; Reroll 5	0.0003373568855	0.0003388926305	0.6485227006	0.3508010499
Flash 6; Reroll 5	0.00003348974568	0.0003512129026	0.7495622212	0.2500530761
Flash 10; Reroll 5	0.0003374569413	0.00001970533470	0.6495348614	0.3501079763
Roll 5, No Flash	0.0001471528100	0.0001489947157	0.7718965269	0.2278073256

Time to Absorption

The final matrix we can compute, as described in [2], is given by $t = Nc$, where N is the fundamental matrix and c is the 17 row column vector in which every entry is 1. By performing this operation, we find a 17 x 1 vector, t, for which every entry represents the expected number of steps before the chain is absorbed when starting in the given transient state s_i. The column matrix t is given below in Table 9. Focusing on the last entry, we can expect the process to take 1.553 steps from any player's initial roll before being absorbed.

Table 9: Expected Number of Steps Before Absorption For Each Transient State.

Flash 2, 3, 4, 6; Reroll 1W	1.821
Flash 5, 10; Reroll 1W	1.511
Flash 2, 4, 6; Reroll 1B	2.132
Flash 3; Reroll 1B	1.777
Flash 5, 10; Reroll 1B	1.821
Flash 2, 3, 4, 6; Reroll 2W	1.689
Flash 5, 10; Reroll 2W	1.502
Flash 2, 4, 6; Reroll 1B, 1W	1.813
Flash 3; Reroll 1B, 1W	1.511
Flash 5, 10; Reroll 1B, 1W	1.564
Flash 2; Reroll 5	3.242
Flash 3; Reroll 5	2.805
Flash 4; Reroll 5	3.242
Flash 5; Reroll 5	3.183
Flash 6; Reroll 5	3.239
Flash 10; Reroll 5	3.187
Roll 5, No Flash	1.553

6 Conclusion

Applying counting techniques and probability to the game Cosmic Wimpout, we found the possible outcomes for the initial roll and calculated the expected score for the initial roll of five dice to be about 26.57 points. Using frequencies and probabilities, we constructed a Markov chain model to represent the game and applied matrix operations to answer questions regarding the duration and probability of the process going through certain states. Specifying conditions in which the player would choose to reroll or not reroll instead of assuming the player only do so when forced to by the rules of the game would provide a more intricate and realistic model for the game and something for future exploration.

Bibliography

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