

Extended Thesis Abstracts

Students' Misconceptions In Middle School Mathematics

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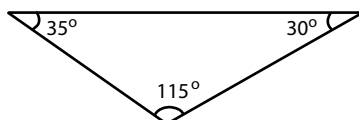


Lindsay Keazer graduated Magna Cum Laude from Ball State in May 2004 with a teaching major in Mathematics. She was named BSU's Outstanding Senior in Mathematics Education for 2003-2004. Lindsay gave a presentation based on her honors thesis at the National Conference on Undergraduate Research in April 2004. Dr. Elizabeth Bremigan was her honors thesis advisor.

Middle school is a significant period in students' mathematical development, a period in which they crystallize their understanding of mathematical concepts and procedures and make the unconscious decision as to whether or not they will be successful in math. This is also a point in students' learning in which misconceptions are born, and if gone unnoticed, these can continue with a student into high school and cause confusion as new concepts are built upon their middle school foundations. However, like an ailment, if caught early these misconceptions can be treated and students' mathematical understanding can be reconstructed. The objective behind my research was to create an informative resource intended for teachers of middle school mathematics, which identifies common mathematical misconceptions exhibited by students. My intention is to equip educators with valuable information so that they can begin to rebuild students' conceptions of specific mathematical content areas and prevent certain misconceptions from occurring or continuing in their students. I collected my data from actual student work on the open response sections of the 2002 sixth and eighth grade ISTEP+ Mathematics Assessments from an Indiana middle school [2, 3]. I analyzed the work of 75 students, 39 sixth graders and 36 eighth graders. Because they are administered near the beginning of each academic year, the ISTEP+ assessments are based on the standards for the previous grade level: in this case, the tests measure fifth and seventh grade

standards. After the ISTEP+ is scored, the results are returned to schools, along with the actual copies of students' written responses from the applied skills section. These sections are returned to schools because they contain valuable information that the scores themselves do not reveal, such as what specific mathematical misconceptions or processes a student is struggling with, or what misconceptions are common among students in a population. However, in order to reach conclusions of this significance from the test data, an extensive analysis is necessary, and most schools do not have the enough people or resources to perform such a study. For that reason I hope that math educators are able to use the results of my research in order to benefit their classroom teaching. From the two assessments, I selected 19 open response tasks to analyze. A unique coding system in the form of a rubric was created for each task, and I applied this rubric in coding both the strategies used and errors made by students. My analysis presents each task, followed by conclusions about misconceptions that clearly existed as well as hints of trouble spots that exist. The tasks and their corresponding misconceptions are categorized according to specific content and process standards as defined by NCTM, and corresponding Indiana Academic Standards are identified. One such task, given to sixth graders and assessing the content standard of geometry, is included below to illustrate the nature of conclusions reached:

Task: Look at the triangle below.



Identify the triangle as right, acute, or obtuse. Write your answer on the line below. Underneath, explain why you identified the triangle as right, acute, or obtuse.

Indiana Academic Standard [1]:

5.4.2 Identify, describe, draw, and classify triangles as equilateral, isosceles, scalene, right, acute, obtuse, and equiangular.

Discussion: From this question it is evident that it is not enough for students to simply be able to classify a triangle, but they must also be able to explain how the triangle is identifiable as such. 44% of students correctly answered that the triangle pictured was obtuse, but less than half of those students were able to explain why they knew it to be an obtuse triangle. Many students labelled every individual angle of the triangle, and some went on to say that it was an obtuse and acute triangle, because both types of angles were present. Others labelled it acute because there were two acute

angles in it. If that were the requirements for an acute triangle, however, obtuse triangles would not exist. A sample student-response follows:

Answer 2 acute, 1 obtuse

[2 acute, 1 obtuse]

I put two acute because an acute angle is smaller than a right angle. I also put one obtuse angle because an obtuse angle is bigger than a right angle. That is how I got my angle!

[I put two acute because an acute angle is smaller than a right angle. I also put one obtuse angle because an obtuse angle is bigger than a right angle. That is how I got my angle!]

Perhaps the misconception here stems from students having already mastered the skill of identifying angles of the triangle as acute or obtuse, but not yet understanding how to use that previous skill in the more complex task of identifying triangles. It can be confusing to students that all the angles of an acute triangle are acute, and yet not all the angles in an obtuse triangle are obtuse (in fact only one angle of an obtuse triangle is obtuse and the other two angles are acute). Having students experiment with drawing triangles and attempting to draw a triangle with more than one obtuse angle could eliminate this misconception.

Because my data comes from one specific school, the results are in some ways singularly representative of this school. However, these results have the potential to be of great use to mathematics educators at other schools. The fact that a misconception exists within some students at one school indicates that there is the possibility of that misconception occurring elsewhere. Thus, the results of this study can be used as a tool to inform educators of mathematical concepts that should be addressed with careful forethought, and to provide them with an awareness of potential misconceptions in order to prevent the creation of these in their classroom.

References

- [1] Indiana Department of Education, *Indiana's Academic Standards—Teacher's Edition: Mathematics*, State Publishing Office (2000).
- [2] Indiana Department of Education, *ISTEP+ Fall 2002 Applied Skills Assessment Book I: Grade 6*, McGraw-Hill (2002).
- [3] Indiana Department of Education, *ISTEP+ Fall 2002 Applied Skills Assessment Book I: Grade 8*, McGraw-Hill (2002).